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GRADUATE WORK IN MATHEMATICS IN UNIVERSITIES AND IN OTHER INSTITUTIONS OF LIKE GRADE IN THE UNITED STATES[1]

I. THE ESTABLISHMENT OF ADVANCED INSTRUCTION IN THE UNITED STATES

FORTY years ago the bachelor's degree granted on the completion of a four years' course of a general character marked not merely the close of a young man's liberal education, but also, except in the case of some lawyers, ministers and physicians, the end of all academic instruction of any kind. In particular, apart from a few exceptional cases, no advanced instruction in mathematics was anywhere provided beyond the usually rather meager ingredients —hardly more than analytic geometry and a little calculus—of this college course, which consisted mainly of prescribed studies. As an external sign of this state of affairs we note that the master's degree, where it existed, was conferred for reasons having very little to do with study, while the doctor's degree was practically non-existent.[2] The desire for higher education in America, which had been felt for many years by some of the leading minds of the country, had been able so far to achieve only momentary and sporadic success.

[1] General report of the committee consisting of Professor Maxime Bôcher, Harvard University, Cambridge, Mass., *chairman;* Professor D. R. Curtiss, Northwestern University, Evanston, Ill.; Professor Percey F. Smith, Yale University, New Haven, Conn., and Professor E. B. Van Vleck, The University of Wisconsin, Madison, Wis. The report is Bulletin No. 6 (1911) of the United States Bureau of Education.

[2] Except at Yale University, where the degree of doctor of philosophy was established in 1860.

The most notable example of such a momentary success, so far as the study of mathematics is concerned, is to be found at Harvard during the fifties and early sixties, where, under the guidance of Benjamin Peirce, a band of young men devoted themselves successfully to the pursuit of higher mathematics.[3] A few of these have since attained world-wide fame, while others were influential in introducing advanced mathematical instruction into the United States twenty or thirty years later. Peirce's success in collecting at this time a fair number of competent students for graduate work seems to have been due primarily to the presence of the office of the American Nautical Almanac at Cambridge from 1849 to 1866, and, secondarily, to the founding in 1847 of the Lawrence Scientific School,[4] which, in those early years, possessed, under the leadership of Louis Agassiz, Jeffries Wyman, Asa Gray and others, some of the aspects of what is now known as a graduate school.

We notice, in passing, the contrast presented at this time, and for many years after, between the increasing supply of good astronomers in this country and the lack of men who, even by a stretch of the imagination, could be called mathematicians. It may fairly be said that the mathematical talent of the country was at this time diverted to astronomy.

Various circumstances united to bring a large measure of success in the establishment of graduate instruction in all fields, and in particular in mathematics, during the years 1870–1890. The great increase of wealth in the country brought with it endowments of many sorts which strengthened the older universities and established some important new seats of learning. Three things may be mentioned which, on this basis of material prosperity, did more than anything else to help forward the cause of graduate study in the critical period we are now considering.

1. *Study Abroad.*—For many years an occasional American had gone abroad to complete his studies. Thus B. A. Gould, a pupil of Benjamin Peirce and a graduate of the class of 1844 at Harvard, who later became eminent as an astronomer, studied with Gauss in Göttingen and took his doctor's degree there in 1848. Similarly J. Willard Gibbs after taking his doctor's degree at Yale in 1863 spent three years (1866–1869) in Paris, Berlin and Heidelberg, where he studied with Kirchhoff, Helmholtz, Weierstrass and others. A few more cases of a similar sort might be recorded, but it was not until the end of the seventies or the beginning of the eighties that the stream of mathematical students from America to Europe (generally to Germany) became a steady one. This tendency to go to Germany for the closing years of study contributed probably more than anything else to build up sound standards of productive scholarship and of graduate teaching, without which all attempts to establish advanced instruction in this country must have remained abortive. Its success was in part due to the establish-

[3] Peirce was tutor or professor of mathematics at Harvard from 1831 till his death in 1880, but, except during the period here considered, it was only in the last ten years of his life that, under the influence of an expanding elective system, he again began to have an appreciable number of advanced students.

[4] In the same year the department of philosophy and the arts was organized at Yale with the purpose of furnishing "resident graduates and others with the opportunity of devoting themselves to special branches of study," these branches embracing "theology, law, medicine and more particularly mathematical science and physical science and its applications." It was in this department that the doctor's degree was established in 1860, as noted above.

ment and the wise administration of traveling fellowships, first at Harvard and then, to a much less degree, elsewhere. We shall return to this important matter of study abroad in a later section.

2. *The Foundation of Johns Hopkins University.*—The magnificent bequest of Johns Hopkins of $3,500,000 for the foundation of a university in Baltimore, and his wisdom in leaving his board of trustees a free hand in the organization of the institution resulted in the adoption, on President Gilman's initiative, of a plan whereby the ordinary undergraduate instruction was relegated to a subordinate position from the very start, so that the new university stood before the American public as the standard bearer of the higher education. This was of inestimable benefit in strengthening the hands of those members of the faculties of the older universities who had been struggling to establish and develop graduate instruction at their own institutions. The presence of the eminent English mathematician, Sylvester, as professor of mathematics during the first seven years of the Johns Hopkins University had also a marked effect in stimulating interest in advanced mathematical studies in America, though it is easy to overestimate his direct influence, as he was a poor teacher with an imperfect knowledge of mathematical literature. He possessed, however, an extraordinary personality, and had in remarkable degree the gift of imparting enthusiasm, a quality of no small value in pioneer days such as these were with us.

3. *The Elective System.*—At the beginning of the period under consideration the lack of students qualified to undertake advanced work was most keenly felt and made any large success in the establishment of graduate instruction an impossibility. The adoption under the lead of President Eliot, first at Harvard and then to a greater or less extent throughout the country, of a far-reaching elective system in the four-years' undergraduate course furnished a possibility for the gradual extension of instruction in the special fields. Without entering on the question of the advantages and disadvantages of the elective system for the college itself, we may safely say that it provided a basis for advanced instruction without which any considerable development of such instruction, at least during the years of which we are now speaking, would hardly be conceivable.[5]

At the close of the period we are considering, when the idea of graduate instruction had already taken a firm hold on many of the stronger institutions of the country, the founding of Clark University exclusively for graduate study in mathematics, psychology, biology, physics and chemistry gave a further impetus to specialization in advanced work, and the opening of the University of Chicago in 1892 may almost be said to mark an epoch in the development of graduate instruction in the west and middle west; for, though that university had from the start an undergraduate department, it stood out, through the character of its faculty and the emphasis laid on research work, as a strong exponent of the graduate idea.

While in these universities, as well as at

[5] Cf., however, the closing remarks of section II. What we desire to emphasize here is that an elective system so arranged as to allow some specialization in individual departments, not merely the choice between various *elementary* subjects, permitted a gradual development of more and more advanced instruction, the students being at first mainly undergraduates. *Such* a development could go on simultaneously at many places, while even a single attempt to duplicate the Johns Hopkins experiment would probably have quickly led to disastrous failure.

Johns Hopkins, advanced instruction was at once placed in a department by itself, in by far the larger number of institutions it developed very gradually within the old college, room being made for it by the elective system; and it was only slowly, even in the larger institutions, that small groups of graduate students began to collect. The somewhat unorganized condition, which was then the rule, is still to be found in the weaker institutions of the country and also in some colleges which in their chosen field of undergraduate work are strong, but which voluntarily renounce any substantial development of graduate instruction. The great universities, however, have all, since the year 1890, developed well-organized graduate schools frequented by the graduates of their own and other colleges. It may be added, to avoid possible misconception, that the graduate schools which sprang full-fledged into existence and those which developed slowly from the old college no longer form two distinct classes. Some of the strongest graduate schools in the country are now to be found among the last-named institutions.

In contrast with such countries as France, Italy and, to a less extent, Germany, we note the complete lack of central control or organization in the United States. Many variations are hereby made possible which are, for a country like ours, almost a necessity; and competition, on the whole healthy, springs up between the different institutions.

In conclusion we note that of late years some technological schools (for instance the Massachusetts Institute of Technology) have undertaken a limited amount of graduate instruction in mathematics. As this instruction does not differ, except in the greater emphasis laid on applied mathematics, from that given at colleges and universities, and since the amount of such instruction at technological institutons is as yet very small in comparison to the whole amount of mathematical graduate instruction in the country, we have not thought it necessary to mention these technological schools specifically in what follows.

II. THE GRADUATE STUDENT OF MATHEMATICS AT THE PRESENT DAY

Owing to the great variety of standards for the bachelor's degree in the different colleges of the country, the students of a single graduate school enter it with very diverse preparation. This is, however, not so disturbing as might be expected, owing to the fact that at every university in which a graduate school exists there is a collegiate or undergraduate department whose instruction is freely open to the graduate student who is in need of it. We may then say that not all work done by graduate students is graduate work. On the other hand, the ambitious and capable senior in colleges allowing considerable freedom of election will frequently be doing work of a distinctly graduate character in the same classes with able graduates of colleges in good standing.

If we thus miss any sharp line of demarcation at the lower limit of the graduate school between graduates and undergraduates, we find a similar phenomenon at the upper limit where the graduate student often passes by almost imperceptible steps into the teacher. Indeed there are graduate schools, even among the better institutions of the country, the bulk of whose students are at the same time assistants or instructors. This, and the very high percentage of graduate students of mathematics the country over who are fel-

lowship and scholarship holders are features of American education which, it is to be hoped, will gradually pass away.[6] They are closely related to the presence in graduate schools of large numbers of students of mathematics who have reached an age when their students days should be over. It can not be too strongly urged on all who give young men advice or who are influential, by awarding scholarships or otherwise, in shaping their careers that it is only in his first youth (not at the age of thirty or thirty-five) that the foundation of real success can be laid by the student of mathematics.

A somewhat different class is formed by school teachers in active service who are at the same time enrolled as graduate students of mathematics, but at any moment take necessarily only a small amount of work. The ambition of these teachers to improve their professional equipment is most laudable. When, however, as is sometimes the case, they form a considerable proportion of the enrollment of a graduate school, they may be a source of weakness to that school in spite of their earnestness of purpose.

The period spent by a student in graduate study varies from one to three, or even more years; and the amount of migration from one university to another does not seem to be large, although the great majority of students attend a graduate school at a different university from that at which their undergraduate years were spent.

We note also that in graduate work coeducation is the almost universal rule, not only in the great state and other western institutions where coeducation forms an integral part of the scheme of education from top to bottom, but even in the most conservative institutions of the east, which do not admit women to their undergraduate departments. Apart from Princeton and the University of Virginia, where no women are admitted, it is only in women's colleges (Bryn Mawr, Vassar, etc.) and in some institutions for men which have held firmly to the undergraduate idea, so that the amount of graduate work is very limited, that one sex alone will be found.

A striking and significant fact is that nearly half of all graduate students of mathematics come from small colleges. This is probably due to the fact that in such colleges students always have the opportunity to study the elements of mathematics and often something beyond the elements, while the inducements for them to turn away into other fields are slight in comparison to those offered at larger institutions where a richer elective system prevails. The tendency so strong in our day and country to regard the man of action as being of nobler clay than the man of thought and ideas, reinforced by the much greater financial prizes open to the former, whether he be lawyer, business man or engineer, creates a situation where it is not easy to secure for mathematical study a due proportion of the strongest youth in our college communities.

III. THE ORGANIZATION OF ADVANCED MATHEMATICAL INSTRUCTION[7]

The purpose of mathematical instruction should be fourfold:

I. To impart knowledge.

II. To develop power and individual initiative.

III. To lead the student to express adequately and clearly what he knows.

[6] Nothing in any way resembling this free award of financial aid is found necessary to induce strong men to attend schools of law or engineering. Cf. the closing lines of this section.

[7] Cf. also the report of subcommittee 1.

IV. To awaken the love of knowledge and to impart scholarly ideals.

The first of these aims, without attention to which the other three can not be obtained, has led to the great development of the lecture system which we find in all graduate schools, and to the use of the textbook and treatise either in connection with these lecture courses or independently of them, and of the original memoir, the reading of which constitutes an art by no means easy to acquire, and which deserves special cultivation at the hands of the members of the teaching staff.

As means used under II. may be mentioned: First, the solution of problems by students either in connection with the lecture courses or in special seminars or proseminars, and secondly, the writing of theses which may or may not be connected with the doctor's degree. This last is also the chief means employed under III., although the quiz (cf. subcommittee 3, section IV.) is sometimes employed effectively for this purpose, and even the brief written problem is not without some value here.

Both II. and III, above should receive more attention than is now commonly given to them, while I. is at present adequately treated, except, perhaps in the relative weakness of applied as distinguished from pure mathematics.

The aim indicated under IV. depends for its attainment less on special methods of instruction and more on the personality of the instructor and his attitude towards science than do I., II. or III. To secure adequately the end in view, an instructor is needed who combines high *scientific* ideals with a commanding or sympathetic personality. Such men could do much to counteract the tendency noted in the closing lines of section II., but, on the other hand, it is precisely this tendency which makes them difficult to secure. Their influence on undergraduate instruction should be no less valuable than in the graduate school.

We must come back once more to the lecture courses which everywhere form the backbone of graduate mathematical instruction. Such a course usually extends either through the whole academic year, that is, from the end of September till early in June, or through the first or second half of this year.[8] The lectures, of somewhat less than an hour each, come usually three times (less frequently twice) a week. While much depends on the local traditions and the personality of the instructor, it may in a general way be said that these lectures have a far less formal character than is the case in European universities. Students will frequently interrupt the lecturer with a question, and short discussions between the instructor and one or more students will not infrequently take place, and at times the more formal quiz (cf. subcommittee 1, section VI., and subcommittee 3, section IV.) finds its place here. Some tact and firmness are occasionally necessary to prevent the loquacious or thick-headed student from monopolizing the time of the class, but on the whole this frequent contact during the lecture between teacher and student is an admirable feature of American higher education, and counteracts, to a certain extent, some evils which usually accompany the lecture system. It is made possible by the smallness of the classes, an audience of twenty-five in a graduate course in mathematics being distinctly unusual.

The range of subjects covered by the courses offered in each graduate school is

[8] Attention must also be called to work in the summer schools and summer quarters. Cf. subcommittee 1, section IV.

very great (cf. subcommittee 1, section V.). This is peculiarly the case in those institutions which have only recently begun a policy of expansion in their graduate work, where the first sign of such expansion often appears in an astounding increase in the number and range of courses offered, for only a small part of which there are students. Indeed, if students should present themselves, the capacity of the teaching force would be completely overtaxed. This is a state of affairs which no self-respecting institution should allow to continue, and there are signs that it is usually of only a temporary nature, since with a real strengthening of the mathematical department of such an institution this inflation tends to disappear. We hasten to add that the stronger institutions, and many smaller institutions with a due sense of proportion, offer admirable selections of courses commensurate with their capacity and the needs of their students, courses which at each institution usually vary considerably from year to year. Even in the weaker institutions where a call for advanced instruction is hardly apparent, it may often be wise to encourage instructors to offer a course of a not wholly elementary character, as it will frequently be found to act as a tonic and, by keeping them in touch with the scientific side of their subject, enable them to make their elementary work more vital.

It was mentioned in section II. that no sharp distinction between graduate and undergraduate work in mathematics can be made. Indeed it is hard to exclude entirely from graduate work anything above the first course in the calculus, now commonly taken in the second undergraduate year. The actual state of affairs is best expressed by regarding the group of courses just following this point, such as a second course in the calculus, the elements of determinants and of the theory of equations, projective geometry, a first course on differential equations, etc., as belonging both to graduate and to undergraduate instruction. From this latter point of view, however, these courses usually appeal only to the student of distinct mathematical ability and seriousness of purpose, whose presence in the course along with graduates does not very greatly affect the character of the course.

As the external signs of success for the graduate student we have the master's and the doctor's degrees. The first of these is commonly given for one year's graduate work done largely in one subject, such as mathematics or physics, and tested either by course examinations in which a higher standard is demanded than is accepted for undergraduates, or by a single examination covering the whole year's work. A thesis is also often required for the master's degree; but the work done on this thesis is not commonly of the nature of research work, and the degree is taken by considerable numbers of students most of whom never proceed further. This degree is given, and properly given, by a large number of institutions, many of which have only a very moderate strength in their graduate mathematical work. Under these conditions suggestions for a minimum standard for the degree are not out of place, and such suggestions will be found in the report of subcommittee 1, section VII.

The doctor's degree originally came to us from Germany, but has long been naturalized and is in all American institutions of good standing distinctly a research degree. In several of our stronger universities it has a standard at least as high as the best German standard. The requirements for the doctor's degree in universities which

have been given to any extent during the last ten years are tolerably uniform (cf. the report of subcommittee 2), but in this matter so much depends on the unwritten standards of individual professors or departments that there still remains a great difference in the ease with which the degree can be obtained at different institutions. It is for this reason that the suggestion which is sometimes made that it would be well to attempt to formulate definite standards for the doctor's degree, to which the universities of the country should conform, seems to be of slight practical value.

In school and college work America adopts in one respect a very different standpoint from France and Germany, and this has a certain indirect influence on graduate work. We refer here to the fact that in the last-named countries a pupil will not be allowed to proceed from class to class, and, particularly, will not be allowed to pass the great educational landmarks (for instance graduation from the gymnasium in Germany) without conforming to a very exacting standard which a considerable percentage of each class fails to attain. In America, on the other hand, the teacher who tries to impede seriously the progress of any but the unusually lazy or stupid soon makes himself impossible. This is not the place to discuss the respective merits of these two points of view in the secondary school or even in the college; but when we come to the graduate student of mathematics it seems clear that the American attitude must be modified, and, as a matter of fact, in all the stronger institutions of the country a much greater ability and earnestness of purpose is demanded for passing examinations and securing degrees in the graduate school than would be allowed to pass muster in undergraduate work. Nevertheless, it is to be hoped that something more will be accomplished in this direction, and that, in particular, candidates for the doctor's degree will be made to feel that success for them at an institution of good standing is not a mere matter of time and patience. It is the more important to insist on this, since, as has just been said, the whole current of secondary and college education runs in another channel.

IV. TEACHERS

We must be concerned with this subject for two different reasons, first, because the great majority of graduate students of mathematics ultimately become teachers in secondary schools, colleges or universities; and, secondly, because on the quality and efficiency of the teachers in the graduate school itself (professors, instructors, etc.) depends to such a large degree the quality of the school.

It is a favorable sign of the gradual elevation of the profession of secondary-school teacher that of late years many persons wishing to adopt this profession spend a year in study in a graduate school. It is true that this time is frequently not spent in the study of a single subject; but for the future teacher of mathematics (or of mathematics and some other subject) to have had a couple of graduate courses in mathematics, usually in the intermediate group, is a very substantial gain over the conditions of twenty years ago. It is to this class of students that the courses on the teaching of mathematics, which are now given at many colleges and universities, mainly appeal.

If we except this group who go into secondary-school teaching, and a second group who study mathematics as a tool for use in some other science, such as physics, it may be said with almost absolute pre-

cision that all other students of mathematics in graduate schools become instructors in mathematics in colleges or universities. The condition of twenty-five years ago, where college instructors in mathematics were taken from among the freshly graduated students of a college (usually the same college where they were to teach), has now become the exception instead of the rule; and where it still occurs, the appointment is usually a temporary one, both the instructor and the college expecting that, after a year or two of teaching, further graduate study will follow. The gain involved in this changed state of affairs, both in breadth of view and in real mastery of the subject, the teaching of which is to be the young man's life work, is so obvious as to require no further comment here. If the student can furthermore be given some comprehension of the fact that the science of mathematics is a living and growing one through contact with other students or instructors who are themselves contributing to this growth, and still more if he himself can take some part in the development of mathematical knowledge, his outlook on mathematics in particular and intellectual life in general will have been so broadened that he can hardly fail to become a better member of a college faculty than would otherwise have been the case.

After all this has been said, we must, however, admit that this question has also another side less pleasant to contemplate. What passes for original research, in this country more even than abroad, is often hardly a real contribution to mathematical progress at all, but merely a grinding out of results, which if they have only never been published before may be as unimportant and unattractive as you please; they form an "original contribution." One is tempted to answer, Yes, in the same sense as the brass button in the contribution box. We may feel certain that in the long run this will be the character of the research work done by students who have no real capacity or inclination for original work, but who are pushed into it by the increasing demand, on the part of certain heads of departments, for the doctor's degree as a necessary preliminary to college teaching. The pressure thus produced will surely, if persisted in, bring forth an increasing yearly crop of doctors—success can be obtained by almost any one with a fair mathematical capacity and with sufficient industry and patience, either by going abroad or by going to one of the weaker American institutions with an ambition for giving the doctor's degree. It is doubtful if the time will ever come, certainly it will not come for a great many years, when all the members of the teaching staffs of the large universities of the country, and the colleges of like rank, can be men with a real capacity for original investigation; the number of all such men in the country falls far short of (one might almost say that it is of a different order of magnitude from) the number of places to be filled.

The pseudo doctor, to whom reference was made above, is often narrowed rather than broadened by the bit of investigation which he has been set to do, and becomes thereby less effective as a teacher, investigation for him becoming a fetich for which he forgets all other ideals. Or, on the other hand, he may let all thought of original work drop out of his mind when once he has secured his degree. In either case the letters he places after his name ought not to go very far in recommending him for teaching positions. A broad and deep mathematical training should surely be demanded by all the institutions of the country which claim collegiate rank as a prerequisite for a permanent appointment on

their teaching staff. They will naturally demand also some ability as a teacher. If in addition they can secure an investigator of a genuine sort, even though his caliber be slight, they should usually regard themselves as fortunate, though a few of the strongest institutions can and should set themselves a much higher standard. On the other hand, our stronger graduate schools should continue, as they are now doing, to encourage every capable student to try his hand at some piece of original investigation, but they should not hesitate, after a fair trial, to tell him, if that turns out to be the case, that he is not fitted for that kind of work.

No specific training for the profession of college or university instructor is commonly given in graduate schools apart from the training in mathematics (cf. subcommittee 3, section V.). The statement made in section III. of the present report that the training in clear and adequate exposition which is given to graduate students of mathematics is frequently insufficient is of peculiar importance in relation to the future teacher. While it is probably not desirable to attempt to train the future college or university instructor in the art of teaching, the question whether more can not be done to lead graduate students of mathematics to express their ideas well both in spoken and in written form is worthy of serious consideration.

Let us turn now from the graduate students, who are to become college instructors, to the actual instructors and professors of mathematics in our colleges and universities. If we compare conditions at the present day with those existing twenty years ago, a very great increase in the standard of mathematical knowledge on the part of the teaching staff is evident. That the improvement here has not been even greater is due in large measure to the fact that the supply of well-trained graduate students falls far short of the demand. Weak appointments are also made from time to time, owing to ignorance on the part of trustees or heads of departments of what really constitutes a mathematician, to the pernicious view that administrative ability may be allowed to take the place of mathematical ability, or to other like causes. Flagrant cases of this kind occasionally occur which make one blush for the good name of American universities, but such cases are now merely sporadic and one gains comfort by contemplating conditions in Germany only a hundred years ago. What is needed here, as in so many other places in American life, is a strengthening of *intelligent* idealism (we have more than enough misdirected idealism amongst us) based upon knowledge, and there seems every reason to hope that the great development of mathematics in this country during the last twenty years, evident chiefly in the growth and activity of the American Mathematical Society, will in an ever-increasing degree supply the intelligent and influential public opinion here needed. The shortage, above mentioned, in the supply of instructors in mathematics forms the most serious aspect of the situation.

For various further points: The excessive burdening of young instructors with drudgery, which still often occurs; inadequate salaries; the burdening of professors with administrative work; we refer to the report of subcommittee 3.

V. STUDY BY AMERICANS ABROAD

No account of higher mathematical education in America would be complete without a reference to the part played by the study of Americans abroad. What an im-

portant factor this was in introducing advanced mathematical instruction and research into America has already been mentioned in section I. In the early days the possibilities for advanced mathematical study in this country were very limited, so that it was natural that students able to do so should go abroad where they could find this opportunity in large measure. At the present day it may safely be said that at several of the stronger American graduate schools most American students find mathematical opportunities better suited to their needs than are to be found at any place abroad. Nevertheless, students still go abroad in apparently undiminished numbers to study mathematics,[9] and their decision to do this is frequently a wise one. Let us inquire how this can be the case.

There come first considerations of a nonmathematical character. It is desirable for every one to become acquainted at first hand with other countries than his own, and this is doubly true for an American, for whom a period of residence in European countries is invaluable. It is true that the student often seems to have brought back from a year or two of residence abroad only a strengthening of his earlier national prejudices, since the mote in the neighbor's eye is so very easy to discern;[10] but if he is worth his salt, he brings with him a fund of impressions and experiences which, as time goes on, greatly enrich his life. For this reason alone study abroad is to be recommended even at some mathematical sacrifice. A second consideration is that the cost of living in Germany, to which country the great majority of students going abroad have always resorted, even after the great increase of the last few years, is still lower than in America, and in particular, the tuition fees are much less than in many of the larger American institutions, especially of the east. These facts largely counterbalance the expense of the trip across the ocean. Finally, it is to be remembered that a year or two of mathematical study in Germany, France or Italy gives the student a reading and speaking knowledge of one of the great languages of modern thought, besides his own native English, such as can hardly be acquired in any other way.

When we come to mathematical considerations, the first question we must ask is whether getting a degree or learning mathematics is the prime object of the student going abroad. It is the former which,

[9] It would be a matter of considerable interest to have statistics on the number of American students who go abroad each year to study mathematics and the length of time they stay. Such statistics would seem to be very difficult if not impossible to secure. As to the proportion of instructors of graduate courses in mathematics who have spent at least one year abroad, see the report of subcommittee 1, section III. Far less important is the question of the number of doctors' degrees conferred on Americans abroad. Such information might be secured. We content ourselves with giving two such items, for which we are indebted to Dr. Dunham Jackson:

At Göttingen in the years 1889–1909, inclusive, twenty-two Americans received the degree in mathematics, while no degrees in mathematics had been conferred on Americans during the four previous years.

At Leipzig in the years 1885–1902, inclusive, eight Americans received the degree in mathematics, while after this time Americans seem to have ceased taking the degree in mathematics there.

At present from two to four Americans take their degree in mathematics in Germany each year, as against an average of sixteen or seventeen in the United States.

[10] There are also cases in which he takes so kindly to foreign conditions as to become out of touch with America. It is, however, rare that this state of affairs should survive his return more than a few months.

owing to circumstances mentioned in section IV., is too often uppermost in his mind. A student of this category had much better go abroad for his degree than to a second-rate American institution. Of course some care must be exercised by him in the choice of his university, or he must have good fortune in writing a thesis whose weak points are not evident on a superficial examination, but his task is, on the whole, not a difficult one, and he gets at least the advantage of a period of foreign residence.

For another class of men foreign study may be recommended without qualification, namely, for able students who have already had a substantial training in one of the better American graduate schools, or who have even taken the doctor's degree at such a school. Such men will naturally go either to one of the great mathematical centers like Paris or Göttingen, where they will have the opportunity to hear lectures by several of the leading mathematicians of the day, and, perhaps, to see some of them occasionally outside of the lecture room; or they will select some mathematician of eminence in a particular field with whom they may hope to gain direct personal contact, and go to the university where he happens to be. Thus of late years a small but steady stream of American students has gone to Italy.

To the students just considered, and to some extent to their weaker comrades mentioned above, the period of residence at a great European mathematical center or of contact with an eminent mathematician at a less important European institution brings with it a realization of what high scientific ideals in mathematics are, and to what an extent they prevail abroad. Such ideals prevail also, it is true, at the strongest American institutions; but it is hard for the young American to appreciate their great diffusion in a ripened civilization until he has experienced it by personal contact.

ADDRESS AT THE UNVEILING OF THE BUST OF WOLCOTT GIBBS IN RUMFORD HALL, CHEMISTS' CLUB, NOVEMBER 25, 1911

BECAUSE of the place of his birth and that where he was educated; because of the profession he chose and which he so highly adorned; because during the greater part of his mature life he applied his splendid talents and broad attainments to the realization of the hopes of the founder of the Royal Institution in his bequests to Harvard College and to the American Academy of Arts and Sciences; and because he was an academician and a club man, it is eminently fitting that the bust of Wolcott Gibbs should be unveiled in the Rumford Hall of the Chemists' Club of the City of New York.

For on February 21, 1822, he was born in this city of New York; in 1841 he received his baccalaureate degree from Columbia College of this city; in 1845 he received the degree of M.D. from the College of Physicians and Surgeons of this city; he chose chemistry as his profession; he was Rumford Professor of Harvard College and Harvard University for forty-five years and a member of the Rumford Committee of the American Academy of Arts and Sciences for thirty years; he was founder, member and president of the National Academy of Sciences; he suggested and organized the Union League Club of New York and he promoted and supported other social organizations.

His education was, however, much broader and more comprehensive than that comprised in his satisfaction of the requirements for the degrees awarded him at Columbia College and the College of Physicians and Surgeons, for in the interim between his graduation from the first named institution and his entrance on the second he served as laboratory assistant to Dr. Robert Hare, professor of chemistry in the University of Pennsylvania, then the most